Reduced order model for accounting for high frequency effects in power electronic components

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This paper proposes a reduced-order model of power electronic components based on the proper orthogonal decomposition. Starting from a full-wave finite-element model and several snapshots/frequencies, the reduced order model is constructed. The characteristic complex impedance can then be extrapolated for the intermediate frequencies with a very low computational cost.

Index Terms—**Reduced-order models, proper orthogonal decomposition, full wave, capacitive effects, finite-element methods.**

I. Introduction

The EVER increasing switching frequencies in modern power electronic converters (from several kHz to several tens of MHz) generate capacitive effects that must be accounted for in early stages of the design for e.g. electromagnetic compatibility issues. In multi-turn windings in DC/DC power supplies, the parasitic capacitance and the leakage inductance present a first resonance around 1 MHz, which is in the operating range of frequencies [1]. Furthermore, at those frequencies the conductors start behaving as transmission lines [1], [2].

In power applications, several approaches have been proposed to approximate the capacitive effects without solving the full-wave Maxwell problem: the coupling of different quasi-static finite-element (FE) formulations [3], or circuit models with parameter extraction based either on the method of moments [2] or on the FE method [1] (see also references herein).

In this paper, we propose a full-wave FE formulation combined with a reduced-order model (ROM) based on the proper orthogonal decomposition (POD). The POD has been widely used in engineering problems [4] and in particular in low-frequency computational electromagnetism (quasi-statics) [5], [7], [6]. It consists in projecting the original basis (constructed using the finite element mesh) onto a reduced basis so that the size of the matrix system is highly reduced. The discrete projection operator is most often determined by means of the snapshot technique [5]. Herein, we solve the full-wave problem in the frequency domain for relevant frequencies in order to obtain the snapshots. The obtained projection operator allows us to dramatically reduce the computational time when solving the problem for intermediate frequencies. As test case we consider the microcoil depicted in Fig. 1.

II. Full wave electromagnetic problem

Let us consider a bounded domain Ω with conducting part Ω_c, non-conducting part Ω_c^C and boundary Γ. Adopting the complex formalism (frequency f, pulsation $\omega = 2\pi f$), the $\boldsymbol{a}-\boldsymbol{v}$ strong formulation of Maxwell's equations and constitutive relations read (complex numbers in bold, $\imath = \sqrt{-1}$):

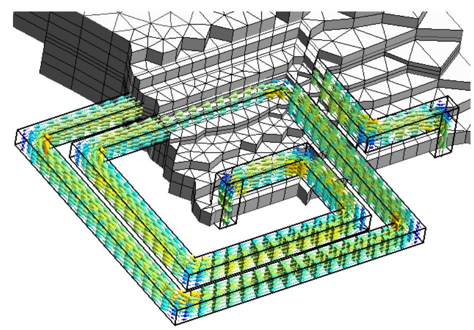

Fig. 1. Microcoil geometry, mesh and induced current density (real part).

$$\operatorname{curl} \nu \operatorname{curl} \boldsymbol{a} - (\omega^2\epsilon - \imath\omega\sigma)\boldsymbol{a} + (\imath\omega\epsilon + \sigma)\operatorname{grad} \boldsymbol{v} = \boldsymbol{j}_s\,, \quad (1)$$

$$\operatorname{div}(\imath\epsilon\omega\boldsymbol{a}) = 0\,, \quad (2)$$

$$\boldsymbol{j} = \boldsymbol{j}_s + \sigma\boldsymbol{e}\,, \quad \boldsymbol{b} = \mu\boldsymbol{h}\,, \quad \boldsymbol{d} = \epsilon\boldsymbol{e}\,, \quad (3)$$

with $\boldsymbol{a}$ the magnetic vector potential, $\boldsymbol{v}$ the electric scalar potential, $\boldsymbol{j}$ the current density, $\boldsymbol{j}_s$ the source current density, $\boldsymbol{e}$ the electric field, $\boldsymbol{b}$ the magnetic flux density, $\boldsymbol{h}$ the magnetic field and $\boldsymbol{d}$ the electric flux density. Material characteristics (linear isotropic media) are the reluctivity $\nu = 1/\mu$ (inverse of permeability), the permittivity ϵ and the conductivity σ. Suitable boundary conditions must be imposed to ensure the uniqueness of the solution [8].

Integration by parts of (1)–(3) yields the weak $\boldsymbol{a}-\boldsymbol{v}$ formulation: find $\boldsymbol{a}$ and $\boldsymbol{v}$ such that

$$(\nu\operatorname{curl}\boldsymbol{a}, \operatorname{curl}\boldsymbol{a}')_\Omega - \omega^2(\epsilon\,\boldsymbol{a}, \boldsymbol{a}')_\Omega + \imath\omega(\sigma\,\boldsymbol{a}, \boldsymbol{a}')_{\Omega_c}$$
$$+(\imath\omega\epsilon + \sigma)(\operatorname{grad}\boldsymbol{v}, \boldsymbol{a}')_\Omega = (j_s, \boldsymbol{a}')_{\Omega_s}, \quad (4)$$

$$(\epsilon\,\boldsymbol{a}, \operatorname{grad}\boldsymbol{v}')_\Omega = 0\,, \quad (5)$$

holds for all test functions $\boldsymbol{a}'$ and $\boldsymbol{v}'$ in suitable function spaces. A Silver-Müller absorbing boundary condition is imposed at the outer boundary Γ. The linear frequency-dependent discretized matrix system can be written as:

$$\mathcal{A}\boldsymbol{x} = \mathcal{J} \quad (6)$$

with $\boldsymbol{x}$ the column vector of N unknowns, $\mathcal{A}$ an $N \times N$ matrix and $\mathcal{J}$ the right-hand-side column vector comprising the source.

III. Model order reduction

We apply the POD [4] to reduce the full-wave matrix system (6). The solution vector $\boldsymbol{x}$ is then approximated by a vector $\boldsymbol{x}_r$ in a reduced basis (size $M \ll N$) such that

$$\boldsymbol{x} \approx \Psi \boldsymbol{x}_r \tag{7}$$

with Ψ a discrete projector operator. This Ψ operator is typically constructed by applying the snapshot technique [5], i.e. generated from original solutions (full-order model) either in the time domain or in the frequency domain. Herein the full problem (6) is solved in the frequency domain for a set of M frequencies (snapshots). The snapshot matrix $\mathcal{S}$ is defined by the column vectors $\boldsymbol{x}_j$, $1 < j \leq M$, the solutions $\boldsymbol{x}$ at the snapshot/frequency f_j. Applying the singular value decomposition (SVD), this snapshot matrix reads

$$\mathcal{S} = \mathcal{V} \Sigma \mathcal{W}^T \,, \tag{8}$$

with $\mathcal{V}$ an $N \times N$ matrix, $\mathcal{W}$ an $M \times M$ matrix and Σ an $N \times M$ diagonal matrix containing the singular values. The i^{th} row of $\mathcal{W}$ represents the entries of the i^{th} column of $\mathcal{S}$ projected in the reduced basis formed by the M columns of the matrix $\mathcal{V}\Sigma$. The operator Ψ is obtained by normalizing the matrix $\mathcal{V}\Sigma$ (or $\mathcal{S}\mathcal{W}$). The reduced system to solve is given by

$$\mathcal{A}_r \boldsymbol{x}_R = \mathcal{J}_r \,, \tag{9}$$

with $\mathcal{A}_r = \Psi^T \mathcal{A} \Psi$ and $\mathcal{J}_r = \Psi^T \mathcal{J}$. Note that computing the SVD of $\mathcal{S}_{N \times M}$ is an expensive (prohibitive) task, in practice, the matrix of correlations $\mathcal{C} = \mathcal{S}^T \mathcal{S} / M$ can be used instead [4].

IV. Application

As test case we consider the micro-coil depicted in Fig. 1. Made of copper, it has 2 turns with a square section $5 \times 5\,\mu\text{m}^2$. The gap between successive wires is $5\,\mu$m. The geometry has been meshed with 13420 prisms and 5360 hexahedra, which yields to 46563 complex unknowns. This discretization is fine enough for the highest considered frequency and it is kept invariant for all computations.

The full-wave problem has been solved with a classical FE approach (reference) and with a RO approach with $M = 2, 3, 6$ snapshots in a wide range of frequencies $f \in [0.01, 100]$ GHz. The modulus and phase of the micro-coil impedance are compared in Fig. 2. The resonance frequency is at 64.2 GHz.

With $M = 2$ snapshots, $f = [0.01, 100]$ GHz, we are not able to recover the impedance value. Adding the resonance frequency to the set of snapshots $M = 3$, $f = [0.01, 64.2, 100]$ GHz, clearly improves the result, getting already a quite accurate approximation. Indeed the average $L2-$relative error on the impedance of 0.0098. With $M = 6$, $f = [0.01, 0.1, 1, 30, 64.2, 100]$ GHz, the FE and the RO curves (both modulus and phase) are indistinguishable, the agreement is excellent with an average $L2-$relative error on the impedance of $6.7e^{-4}$.

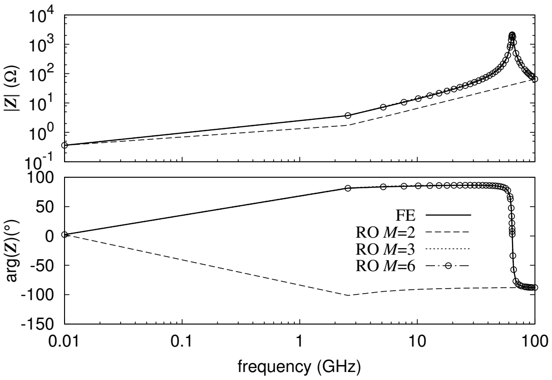

Fig. 2. Modulus (up) and phase (down) of the micro-coil impedance as a function of frequency. Comparison of ROM results (different number of snapshots $M = 2, 3, 6$) with reference FE solution.

With regard to the computational time, a full FE solution requires ≈31.1 s while a ROM solution is obtained after ≈0.0017 s. The gain in computational cost is thus huge for determining the impedance values for intermediate frequencies, provided well-chosen snapshots.

V. Conclusion

In this paper, we proposed a POD-based model-order reduction approach to fully characterize power electronic components in a wide frequency range with a very low computational cost. Using a set of well chosen frequencies/snapshots, we are able to compute the characteristic impedance (modulus and phase) for the whole spectrum.

In the extended paper, the proposed approach will be elaborated in detail and further analysis of the ROM results performed, paying particular attention to the numerical stability, accuracy and effectiveness with the choice of frequencies.

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References

[1] Z. De Grève, O. Deblecker, J. Lobry, “Numerical modeling of capacitive effects in HF multi-winding transformers–Part II: Identification using the finite-element method,” *IEEE Trans. Magn.,* vol. 49, no. 5, pp. 2021–2024, 2013.

[2] J. Zwysen, P. Jacqmaer, R. Gelagaev, J. Driesen “An electromagnetic circuit simulator for power electronics,” *IEEE Trans. Magn.,* vol. 48, no. 2, pp. 799–802, 2012.

[3] P. Dular, R.V. Sabariego, P. Kuo-Peng, “Three-dimensional finite element modeling of inductive and capacitive effects in micro-coils,” *COMPEL: The International Journal for Computation and Mathematics in Electrical and Electronic Engineering,* vol. 25, no. 3, pp. 642–651, 2006.

[4] W. Schilders, H. V. der Vorst, J. Rommes, *Model order reduction: theory, research aspects and applications,* Springer-Verlag, 2008.

[5] Y. Sato, H. Igarashi, “Model reduction of three-dimensional eddy current problems based on the method of snapshots,” *IEEE Trans. Magn.,* vol. 49, no. 5, 1697–1700, 2013.

[6] T. Henneron, S. Clénet, “Model order reduction of multi-input non-linear systems based on POD and DEI methods,” *IEEE Trans. Magn.,* vol. 50, no. 2, pp. 7000604-1–4, 2014.

[7] D. Schmidthäusler, M. Clemens, “Low-order electroquasistatic field simulations based on proper orthogonal decomposition,” *IEEE Trans. Magn.,* vol. 48, no. 2, pp. 567570, 2012.

[8] R. Hiptmair, F. Krämer, J. Ostrowski, “A robust Maxwell formulation for all frequencies,” *IEEE Trans. Magn.,* vol. 44, no. 6, pp. 682–685, 2008